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AN
INTRODUCTION
TO
PLANE TRIGONOMETRY,
WITH ITS
APPLICATION
TO
Altimetry and Longimetry.

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DESIGNED FOR THE USE OF SCHOOLS.

By *WILLIAM WRIGHT,*
TEACHER OF THE MATHEMATICS.

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CONTENTS.

PART I.

Page.
Geometrical Definitions 5
Geometrical Problems 8
Geometrical Theorems 14
Uſeful Remarks on Triangles 22
The Conſtruction of Lines upon the Plane Scale 27

PART II.

Plane Trigonometry 31
Reſolution of Right-angled Plane Triangles . 41
Reſolution of Oblique-angled Plane Triangles 50

PART III.

Altimetry 67
The Application of Trigonometry to Altimetry and Longimetry } 67
To take an Angle of Elevation with the Quadrant } 68

To

Page.
To take an Angle of Depreſſion 69
To take an Horizontal Angle with the Protractor 69
Altimetry exemplified 70
Longimetry exemplified 81
Promiſcuous Queſtions 96
Table of Rhumbs 110
Table of Engliſh Lineal Meaſures . . . 111
Table ſhowing the Height of the Apparent Level above the True, for every Hundred Yards Diſtance upon the Earth's Surface, on the one Hand, and for every Mile on the other . 112
Application of the above Table 113

MATHE-

MATHEMATICAL CHARACTERS.

=	implies, or ſtands for,	Equality.
+		Addition, thus $2 + 4 = 6$.
−		Subtraction, as $4 - 2 = 2$.
×		Multiplication, ſo $4 \times 2 = 8$.
÷		Diviſion, as $4 \div 2 = 2$.
△		A Triangle.
□		A Square.
▭		A Rectangle.
<		An Angle.
∥		Paralleliſm.
⊥		Perpendicular.
∟		A Right Angle.
√		Square Root.
S.		Sine.
Co. S.		Co-ſine.
T.		Tangent.
Co. T.		Co-Tangent.
Sec.		Secant.
Co. Sec.		Co-Secant.
Co. Ar.		Complement Arithmetic.
: :: :		Proportion, thus, as 2 : 4 :: 6 : 12; and is read, as 2 is to 4 ſo is 6 to 12.
°		Degrees.
′		Minutes.
″		Seconds.

GEOMETRICAL DEFINITIONS.

PART I.

1. *GEOMETRY* is a ſcience which treats of the extenſion of quantities in general.

FIG. I.

2. *A Line* is *length*, and is generated by the *motion of a point*, whether it be ſtraight or curved, as A B, and C D.

FIG. 2.

3. *Parallel Lines* are thoſe which are always *equidiſtant*, or, if *infinitely* produced, would never meet; as E F, G H.

FIG. 3.

4. *A Plane rectilineal Angle* is conſtituted by the *inclining*, or *opening* of two right lines, which meet in one and the ſame point I.

Note. When three letters are uſed to expreſs an angle, that which ſtands at the angular point is read in the middle; and the other two ſtand upon the lines which form the angle; thus for the angle at I, (Fig. 3.) we ſay the angle K I L.

FIG. 4.

When a right line M P meets with another O N in ſuch poſition, that the angles O P M and M P N are equal to each other; then is M P ſaid to be *perpendicular* to O N, and thoſe angles thus formed are called *right angles*.

FIG. 5.

6. *An obtuſe angle* is *more* than a right one, as Q R S.

FIG. 6.

7. *An acute angle* is *leſs* than a right one, as T U V.

FIG. 7.

8. *A Triangle* is a figure bounded by three lines, and conſequently contains as many angles, as W X Y.

FIG. 8.

9. *A right angled Triangle* is ſuch as has one of its angles a right angle; or, which is the ſame, when two of its ſides fall perpendicularly to each other, as A Z B.

GEOMETRICAL PROBLEMS.

PROBLEM I.

FIG. 9.

To biſect a given finite right line A B.

1. From the point A as a centre, and with any radius greater than half A B deſcribe an arch.

2. From B with the ſame radius deſcribe another arch, which will interſect the former in C and D.

3. Laſtly, through the points C and D, draw the line C E D, and it will biſect A B in E.

PROBLEM II.

FIG. 10.

From a given point F, at or near the middle of a given right line G H, to erect a perpendicular.

1. On each ſide of the given point F, ſet off two equal diſtances F I and F K.

2. From I and K as centres, and with any radius greater than half I K, deſcribe arches interſecting each other in the point E.

3. Draw E F, and it will be the perpendicular required.

PROBLEM III.

FIG. II.

To erect a Perpendicular from the Extremity of a Line.

1. From the point O with any radius, deſcribe the arch Q N M.

2. Then, from the point Q, with the ſame diſtance, turn the compaſſes twice over upon this arch.

3. And from N and M with the ſame radius, deſcribe arches interſecting in L.

4. Through L, draw L O, and it will be the perpendicular required.

PROBLEM IV.

FIG. 12.

From a given Point R, diſtant from a given Line S T, to demit a Perpendicular.

1. From the given point R, draw any line R W to meet S T in any point W.

2. Biſect W R in V, and with the radius V W equal V R, deſcribe an arch cutting S T in X.

3. Laſtly, draw R X, and it will be the perpendicular required.

PROBLEM V.

FIG 13.

To make an Angle to contain a given Number of Degrees.

1. From a ſcale of chords, take the chord of 60 degrees in your compaſſes,

and with one foot in Y, deſcribe the arch C B, cutting Y A in C.

2. From the ſame ſcale of chords take the propoſed number of degrees, (as ſuppoſe 30°). and apply them from C to B.

3. Thro' B draw Y Z, and the angle Y will contain 30 degrees.

Note. An obtuſe angle is generally taken off at twice.

Sometimes the angle is formed by any divided arch or inſtrument, by applying its centre to the point Y, and its radius along the line Y A, then make a mark at B, at the propoſed number of degrees, through which draw Y Z, which is the ſame as above.

PROBLEM VI.

FIG. 14.

Through a given point D, *to draw a line parallel to a given line* E F.

1. From any point G in the line E F, and with the radius G D, deſcribe the arch D H.

2. And with the ſame radius, and one foot in D, ſtrike the arch G I.

3. Take the arch D H in the compaſſes, and apply it from G to I.

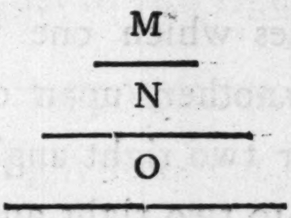
4. Through the points D and I draw K L, and the problem is effected.

Note. This problem is more readily ſolved by means of a parallel ruler.

PROBLEM VII.

FIG. 15.

To lay down a Triangle, whoſe three Sides ſhall be reſpectively equal to three given Right Lines M, N, O.



1. Draw the line R Q equal to one of the given lines O.

2. From R, with a radius equal to N, deſcribe an arch.

3. From Q with the radius M, deſcribe an arch interſecting the former in P.

4. Laſtly, from P, draw P Q, R P, and it is done.

Note. The three lines propoſed muſt be of ſuch length, that any two of them taken together, will be greater than the third, otherwiſe the problem is impoſſible.

GEOMETRICAL THEOREMS.

TREOREM I.

The angles which one right line makes with another, upon one ſide of it, are either two right angles, or two angles equal to two right angles.

FIG. 16.

1. Let B E be perpendicular to A D, then by Def. 5, when the adjacent angles which one ſtraight line makes with another are equal, the angles are called right angles; therefore A E B, D E B, are each of them right angles.

2. If the line be not perpendicular but, obliquely, as E C, then, A E C and D E C are together equal to A E B, and D E B, for the angle B E C which is added to one right angle is ſubtracted

from the other, therefore the whole muſt be the ſame.

Cor. From this it is manifeſt, that all the angles which can be made at any point, on the ſame ſide of a right line, are equal to two right angles.

THEOREM II.

If two right lines cut one another, the vertical, or oppoſite angles ſhall be equal.

FIG. 17.

For by Theor. 1, I K F and F K G are together equal to two right angles, alſo F K I and I K H are together equal to two right angles, but I K F is common to both; therefore F K G is equal to I K H.

Cor. 1. Hence it appears, that if two right lines cut one another, the angles they make at the point of interſection, are together equal to *four right angles.*

Cor. 2. And conſequently that all the angles made by *any number* of right lines interſecting in one point, are together equal to *four right angles.*

THEOREM III.

If a ſtraight line fall upon two parallel right lines, it ſhall make the alternate angles equal to one another, and the exterior angle equal to the interior and oppoſite one upon the ſame ſide, and alſo the two interior angles upon the ſame ſide equal to two right angles.

FIG. 18.

Since the line S M is parallel to the line R N, the one will have the ſame inclination to the line L Q, as the other has; therefore the angle N P O is equal to the angle L O M; therefore, by Theor. 1, the angle M O P is equal to the angle R P O.

Alſo, by Theor. 2, the angle M O P is equal to the angle S O L, and conſequently S O L is equal to R P O; add to each of theſe the angle S O P, then the angles S O L and S O P are together equal to the angles S O P and R P O; but, by Theor. 1, the angles S O L and S O P are together equal to two right angles; therefore S O P and R P O are together equal to two right angles.

THEOREM IV.

If any ſide of a plane triangle be produced, then will the exterior angle be equal to the ſum of the two interior and oppoſite ones; and the ſum of the three interior angles of any plane triangle is equal to two right angles.

FIG. 19.

Let V Z be parallel to T Y; then, becauſe theſe two lines are parallel, and are cut by T Z, the alternate angles Y T Z and T Z V are equal, by Theor. 3. Alſo, becauſe T Y is parallel to V Z, and W Y falls upon them, the angle W Z V is equal to the angle Z Y T; therefore the whole external angle T Z W is equal to the ſum of the two internal and oppoſite angles Y and T. By Cor. Theor. 1, the angles at the point Z are together equal to two right angles, but the angle T Z W is equal to the ſum of the two angles T and Y; therefore the angles T Y Z, Z T Y, and T Z Y are together equal to two right angles.

THEO-

THEOREM V.

In any right angled triangle, the ſquare which is deſcribed upon the ſide ſubtending the right angle, is equal to the ſum of the ſquares deſcribed upon the ſides which contain the right angle.

FIG. 20.

Draw C H parallel to K I, join C I and C G; the angle I K F is equal to A K C; to each of them add C K F, then is C K I equal to A K F, and becauſe A C and K G are ſquares, C K muſt be equal to K A, and K I equal to K F, therefore (by *Eucl.* iv. 1.) the triangle C K I is equal to the triangle A K F. But (by *Eucl.* xli. 1.) the rectangle K H is equal to twice the triangle C K I, and the parallelogram F H equal to twice the triangle C F G, alſo the ſquare A C is double the triangle

A K F, and the ſquare C E double the triangle E F K; but the triangle A K F is equal to the triangle C K I, and the triangle E F K to the triangle C F G; therefore the ſquare A C is equal to the rectangle K H, and the ſquare C E equal to the rectangle F H, and conſequently the ſquares A C and C E are together equal to the ſquare K G.

Cor. 1. Hence, in a *right angled* triangle, the ſquare of the hypothenuſe, leſs by the ſquare of one of the ſides including the right angle, is equal to the ſquare of the other ſide.

Cor. 2. The rectangle made by the ſum and difference of the hypothenuſe and one ſide, is equal to the ſquare of the other ſide.

THEO-

THEOREM VI.

If a line be drawn parallel to one of the ſides of a triangle, it will cut the other ſides proportionally.

FIG. 21.

Draw C D parallel to A E, and join C E and D A; then (by *Euc.* xxxvii. 1.) the triangle C D E is equal to the triangle C A D. Therefore as the triangle B C D : the triangle C A D :: B C : C A, and as the triangle B C D : the triangle D C E :: B D : DE; then by cancelling the two firſt terms, we ſhall have as B C : C A :: B D : D E.

Cor. The correſponding ſegments of the triangle cut off by the line drawn parallel to the ſide, are proportional to one another, and to the whole ſides, and the triangle thus cut off is ſimilar o the whole triangle.

SOME USEFUL REMARKS ON

TRIANGLES.

1. In every plane triangle, the longeſt ſide always ſubtends the greateſt angle, and the ſhorteſt ſide the leaſt: conſequently if two ſides be equal, their oppoſite angles are equal; and if the triangle be equilateral, it will be equiangular.

2. Any ſide of a plane triangle is leſs than the ſum, but greater than the difference of the other two ſides.

3. If a perpendicular be demitted from the vertex of a triangle upon the longeſt ſide or baſe, then the difference of the ſquares of the ſides is equal to the difference of the ſegments of the baſe.

4 In an iſoſeles triangle, if a line be drawn from the vertical angle to *any*

point in the baſe; then will the ſquare of *that* line, together with the product of the ſegments of the baſe, be equal to the ſquare of the ſide.

5. If one angle of a triangle be equal to 120°, then the ſquare of the baſe will be equal to the ſum of the ſquares of both the ſides, together with the rectangle of thoſe ſides; alſo, if thoſe ſides be equal to each other, then will the ſquare of the baſe be equal to three times the ſquare of one ſide, or equal to twelve times the perpendicular demitted from the vertex to the baſe.

6. Any triangle may have a circle circumſcribed about it, touching all its angular points, and a circle inſcribed touching all its ſides.

7. The ſquare of the ſide of an equilateral triangle is equal to three times the ſquare of the radius of its cricumſcribing circle.

8. If the three angles of one triangle be equal to the three angles of another, each to each; then thoſe two triangles are ſimilar each to the other, and their like ſides are proportional.

9. Triangles that have any three parts of the one (excepting the three angles) equal to the three correſponding parts of another, each to each; then thoſe two triangles are not only ſimilar, but have alſo their ſix correſponding parts equal.

10. Three perpendiculars beſecting the three ſides of a triangle, all meet in one point, and that point is the centre of the circumſcribing circle.

11. Three lines beſecting the three angles of a triangle all meet in the ſame point, and *that* point is the centre of the inſcribed circle.

12. Three perpendiculars drawn from the three angles of a triangle, to the oppoſite ſides, interſect in one point.

13. If the vertical angle of a triangle be beſected by a line cutting the baſe; then the ſegments of the baſe are proportional to their adjacent ſides.

14. The ſquare of the line beſecting the vertical angle of a triangle, is equal to the difference between the rectangle of the ſides, and the rectangle of the ſegments of the baſe.

15. In a plane triangle, when a line is drawn from the vertical angle to biſect the baſe; then the ſum of the ſquares of the ſides, is equal to twice the ſum of the ſquares of half the baſe and the line beſecting it.

16. The angle which is formed by the perpendicular from any angle, and the line drawn from the ſame angle to the middle of the oppoſite ſide, is equal to half the difference of the angles at the baſe.

17. Triangles which ſtand upon the

ſame baſe, and between the ſame parallels, are equal to one another.

18. Triangles upon the ſame baſe are to one another as their altitudes; and triangles of equal altitudes, are to one another as their baſes; alſo triangles which are equal, have their altitudes and baſes mutually equal.

19. Every triangle is equal to half the parallelogram of the ſame baſe and altitude with it.

Note. The demonſtrations of moſt of theſe uſeful properties may be found in *Euclid's Elements*.

The fundamental Conſtruction of ſuch Lines upon the Plane Scale, as are commonly uſed in Trigonometry, Navigation, Aſtronomy, Dialling, &c. &c.

FIG. 22.

1. WITH any convenient radius deſcribe a circle, and quarter it by drawing D C and B A to interſect each other at right angles in E, and produce the diameter C D towards G, and draw the line A F parallel to it; alſo join the extremities of the two diameters by the lines D A, A C, C B, and B D.

2. Divide the arch A D into 90 equal parts, and on A as a centre, with the compaſſes transfer theſe equal diviſions to the line A D, to which annex the correſponding numbers, and it will be a line of chords to be transferred to the ſcale.

3. For the line of ſines, draw lines parallel to the radius D E, through each of the diviſions upon the arch A D, and they will divide the radius E A into *ſines*, or *verſed ſines*, obſerving that it muſt be numbered from E to A for the *ſines*, but from A to E for the *verſed ſines*.

4. Lay a ruler from the centre E to the ſeveral diviſions on the arch D A, and it will cut the line F A, which will be a line of tangents, and is numbered from A to F with 10, 20, &c.

5. Set one foot of the compaſſes in the centre E, and extend the other to the ſeveral diviſions 10, 20, &c. on the line A F, and transfer theſe diviſions ſeverally to the line D G, which will become a line of ſecants, and muſt be numbered from D to G with 10, 20, &c.

6. The line of ſemitangents is conſtructed by laying a ruler from B to each of the diviſions on the arch D A,

which will interſect the radius D E in the diviſions of the ſemitangents, which are to be numbered with the correſponding figures in the arch D A.

7. Divide the arch B C into eight equal parts, and transfer each of theſe parts ſeverally to the chord B C, and it will be the line of rhumbs.

8. For the line of longitudes, divide the radius B E into 60 equal parts, draw lines from each of theſe diviſions parallel to the radius E D, and they will interſect the arch B D in as many points; then with one foot in B, tranſfer each of theſe diviſions to the chord B D, and it will give the diviſions of the line of longitudes.

9. Laſtly, for the line of latitudes, apply a ruler from D, to the ſeveral diviſions on the ſines upon A E, and it will interſect the arch A C in as many points; then from C as a centre, trans-

fer each of theſe points ſeverally from the arch to the chord A C, and it will be the line of latitudes.

PLANE TRIGONOMETRY.

PART II.

DEFINITIONS.

1. *TRIGONOMETRY* is that art, by which we compute the ſides and angles of a *triangle* which are unknown, from others which are known or given.

2. The *circumference* of every circle is conceived to be divided into 360 equal parts, called *degrees*, each degree into 60 equal parts, called *minutes*, each minute into 60 equal parts, called *ſeconds*, and ſo on in *ſexagenary* order.

FIG. 23.

3. The *meaſure* of any angle of a plane triangle, is the arch of a circle intercepted between the two lines that conſtitute that angle; ſo that the arch B C is called the meaſure of the angle B A C, C D, that of D A C, and B E that of E A B, and ſo many degrees, &c. as are contained in each of theſe arches, ſo many degrees, &c. does each of theſe angles contain; but the ſides of a plane triangle are meaſured by the equal parts of ſome ſtraight line or ſcale, conſidered as a ſtandard meaſure; as a *pole*, a *yard*, a *foot*, an *inch*, *&c.*

4. Since *all circles*, whether *great or ſmall*, have their peripheries divided into the ſame number of equal parts; it appears that whatſoever be the radius of the circle of which the meaſure of a given angle is an arch, that arch will

contain the ſame number of degrees, &c.

5. In a right angled triangle, the ſide ſubtending the right angle is called the hypothenuſe, and the two ſides, including the right angle, are called the baſe and perpendicular; and the ſum of the ſquares of the baſe and perpendicular is equal to the ſquare of the hypothenuſe.

6. Since the ſum of the three angles of any plane triangle, is equal to two right angles or 180°; therefore, if one of the angles in a right angled triangle be ſubtracted from 90°, the remainder will be the other; and the ſum of any two angles in either an oblique, or right angled triangle being ſubtracted from 180°, the remainder will be the third angle.

7. The reſolution of triangles is founded on the *mutual proportions* which

ſubſiſt between their *ſides* and *angles*; theſe proportions are known by finding the relations between the radius of a circle and certain other lines drawn within and without it.

FIG. 24.

8. The *ſupplement* of an arch is what it wants of a ſemicircle, or 180°; ſo K E G is the ſupplement of G A.

9. The *complement* of an arch is what it wants of a quadrant, or 90°: ſo E G is the compliment of G A.

10. The *ſine* of an arch is a right line drawn from one of the extremities perpendicularly to the diameter which terminates in the other; ſo G B is the ſine of G A.

Cor. The ſine of a *quadrant*, or 90°, is equal to the radius.

11. The *verſed ſine* of an arch, is the ſpace contained between the ſine and

the circumference, and it is meaſured upon a line paſſing through the centre of the circle; ſo B A is the verſed ſine of G A.

12. The *tangent* of an arch is a right line which touches the circle in one extremity of that arch, and is cut by a right line drawn from the centre through the other extremity of the arch; thus A H is the tangent of G A.

13. The *ſecant* of an arch is a right line drawn from the centre through one end of the arch, and produced till it interſect the tangent of the ſame arch; thus, H C is the ſecant of G A.

Cor. To def. 10, 12, 13, the *ſine tangent* and *ſecant* of any arch G A, are always the *ſine*, *tangent*, and *ſecant* of its ſupplement G E K.

14. The *coſine*, *coverſed ſine*, *cotangent*, and *coſecant* of any arch, are always the *ſine*, *verſed ſine*, *tangent*, and *ſecant* of the complement of that arch; thus, G D is the coſine, E D the coverſed ſine, E F the cotangent, and F C the coſecant of the arch G A.

15. The *ſine*, *tangent*, and *ſecant* of any arch, are the ſine, tangent, and ſecant of any *angle*, of which that arch is a meaſure.

16. The chord of an arch G S is a right line G S drawn between the extremities of the arch.

17. The *proportion* of the ſines, tangents, ſecants, &c. to their radius is ſometimes expreſſed in natural numbers, which make what are called the tables of natural ſines, tangents, ſecants, &c. So if the angle A C G be 63° 15′.

Then the radius C A=C G being 1·0000000
the ſine will be - G B= ·8929789
the tangent - - H A=1·9839636
the ſecant - - - C H=2·2217362
And the ſine of the complement
D G=26° 45′ - - - ·4500984
the cotangent - - - E F= ·5040415
the coſecant - - - C F=1·1198472

And by this artifice we have always a triangle ready calculated in the tables, and ſimilar to *any one* that can be propoſed for ſolution.

18. Sometimes the proportion of ſines, tangents, ſecants, &c. is expreſſed in *logarithms*, which are the logarithms of the above natural numbers, and theſe logarithms conſtitute the tables of artificial ſines, tangents, ſecants, &c.

19. The moſt conſiderable properties and relations of ſines, tangents, ſecants, &c. are deducible from the ſimi-

lar triangles they compoſe in the figure: Thus the ſimilar triangles G C B and H C A give G B : B C :: H A : A C; that is, The ſine is to the coſine, as the tangent is to the radius.

20. Again, in the ſame triangles, we have G B : H A :: C G : C H; that is, The ſine is to the tangent, as the radius is to the ſecant.

21. Alſo we have, C B : C A :: C G : C F; that is, The coſine is to the radius, as the radius is to the ſecant.

22. Again in the ſimilar triangles H A C, C E F, as H A : A C :: C F : E F; that is, The tangent is to the radius, as the radius is to the cotangent.

23. And the triangles G B C, C E F give G B : C G :: C E : C F; that is, The ſine is to the radius, as the radius is to the ſecant.

In ſolving queſtions in trigonometry, we ſhall uſe three methods, viz. 1. Geometrical Conſtruction—2. Arithmetical Calculation—and 3. Inſtrumental Operation.

When the triangle in queſtion is to be conſtructed, it is done by laying down the ſeveral parts and magnitudes given; the ſides from a ſcale of equal parts, and the angles from a line of chords, or any other ſuitable inſtrument; which done, the unknown parts are meaſured by the ſame ſcales, and by theſe means they become known.

When the queſtion is to be ſolved arithmetically, then after the three given terms are ſtated according to the following rules; which terms when we uſe the natural numbers, will conſiſt partly of the numbers on the given ſides, and partly of the ſines, &c. of the given angles; then like all other proportions in which three terms are given to find a fourth; multiply the ſecond and third terms together, and divide the product by the firſt, and the quotient will be the fourth term.

When the artificial logarithms are uſed, the operation is performed by adding the logarithms of the ſecond and third terms together, and from the ſum ſubtracting the logarithm of the firſt, and the remainder is the logarithm of the fourth term; and this method is preferable to the former, by reaſon of the long and tedious multiplications and diviſions attending the natural ſines, tangents, and ſecants.

To perform a caſe inſtrumentally, ſuppoſe upon Gunter's lines, with one foot of the compaſſes in the firſt term, extend the other to either the ſecond or third which happens to be of the ſame name with the firſt, then that extent will univerſally reach from the other to the fourth.

Note. When Gunter's lines are uſed, for the *ſides* of triangles, we uſe the *line of numbers*, marked *Num.* but for the *angles* we uſe the *line* of *ſines*, *tangents*, or *ſecants*, marked *Sin Tan* and *Sec.* according as the rule for the proportion directs.

Reſolution of Right angled Plane Triangles.

In any right angled plane triangle, if the hypothenuſe be made radius, then the other two ſides become the ſines of their oppoſite angles; but if either of the ſides be made radius, then the other ſide becomes the tangent of its oppoſite angle, and the hypothenuſe the ſecant of the ſame angle. Hence theſe

THEOREMS.

1. As one ſide given - - :
   Is to the ſine of its oppoſite angle ::
   So is another ſide given :
   To the ſine of its oppoſite angle.

And the contrary.

2. As the radius - - :
   Is to either leg given - ::
   So is the tangent of its adjacent angle :
   To the other leg, and ::
   So is the ſecant of the ſame angle :
   To the hypothenuſe, and *vice verſa.*

Hence it appears, that to find an angle the proportion muſt begin with a ſide; but to find a ſide, it muſt begin with an angle.

Note. In every triangle, there muſt be three of its parts given, one at leaſt of which muſt be a ſide.

EXAMPLES.

FIG. 25.

1. In the right angled triangle A B C, there are given the baſe A C 170, and the angle A 29° 30′; to find the perpendicular, and the hypothenuſe.

GEOMETRICALLY.

1. Draw A C, and lay of thereon 170, from any ſcale of equal parts.

2. On C erect the perpendicular C B.

3. Make the angle A equal to 29° 30′.

4. Join A B and it is done.

Then the required parts being meaſured by the ſcale of equal parts will be found to be as follow: viz. B C =96·18. and A B=195·3.

ARITHMETICALLY.

The triangle ſimilar to the propoſed one, is the tabular triangle Abc; which has the ſame angles, viz. A=29° 30′, and b=60° 30′: then by making Ab radius, we have by the table of natural ſines, bc=4924236, and Ac=8702124; whence this analogy:

$$\text{As}\begin{cases}\text{Ac} : \text{bc} :: \text{AC} : \text{BC}\\ 8702124 : 4924236 :: 170 : 96{\cdot}18134\end{cases}$$

The analogy for the hypothenuſe will be:

$$\text{As}\begin{cases}\text{Ac} : \text{Ab} :: \text{AC} : \text{AB}\\ 8702124 : 1{\cdot}0000000 :: 170 : 195{\cdot}322\end{cases}$$

			Logarithms.
As ſi. angle B	- -	60° 30′ -	9·9396968
Is to the baſe A C	-	170 -	2·2304489
So is ſi. angle A	-	29° 30′ -	9·6923388
To the perpendicular BC		96·18134	1·9830909

And,

As ſi. angle B - - -	60° 30' -	9·9396968
Is to the baſe A C - -	170 -	2·2304489
So is ſi. < C - - -	90° 00'	10·0000000
To the hypothenuſe AB	195·322	2·2907521

But if Ac be made radius, then Ac and bc, which were ſines before, will become radius and tangent, and Ab ſecant: hence we have theſe analogies:

$$1.\ \text{As}\left\{\begin{array}{l}\text{Ac} : \text{cb} :: \text{AC} : \text{CB} \\ 1\cdot0000000 : 5657728 :: 170 : 96\cdot18134\end{array}\right.$$

And,

$$2.\ \text{As}\left\{\begin{array}{l}\text{Ac} : \text{Ab} :: \text{AC} : \text{AB} \\ 1\cdot0000000 : 1\cdot1489555 :: 170 : 195\cdot322\end{array}\right.$$

		Logarithms.
As radius - - - - - - - -		10·0000000
Is to the baſe C A - -	170 -	2·2304489
So is tang. angle A - -	29° 30' -	9·7526420
To the perpendicular BC	96·18134	1·9830909

And,

As radius - - - - - - - - -		10·0000000
Is to the baſe C A - -	170 -	2·2304489
So is ſecant angle A -	29° 30' -	10·0603032
To the hypothenuſe AB	195·322	2·2907521

INSTRU-

INSTRUMENTALLY.

1. Extend the compaſſes from 60° 30′ to 29° 30 on the line of ſines, and that extent will reach from 170 to 96·2 the perpendicular, on the line of numbers.

2. Extend from 60° 30′ to 90° 00′ on the line of ſines, and that extent will reach from 170 to the hypothenuſe 195·3 on the line of numbers.

2. In the right angled triangle A B C, there are given the hypothenuſe A B 560 yards, and the angle A 37° 2c′; to find the baſe and perpendicular.

Anſ. Baſe 445·2676 yards.
Perp. 339·6125 yards.

3. If in the triangle A B C, the hypothenuſe be 275, and the baſe 195, what are the angles and perpendicular?

° ′ ″
Anſ. The angle A 45 9 39.
——— C 44 50 21.
Perpendicular 193·9077.

4. In the triangle A B C, there are given the hypothenuſe A C 146·5, and the angle B 36° 13′, to find the baſe and perpendicular.

Anſ. { Baſe 86·29200
Perp. 117·8314

5. In the triangle D E F, are given, the perpendicular E F 116·71 miles, and the angle D 56° 45′, to find the baſe and hypotheneuſe.

Anſ. { Baſe 76·51843 miles.
Hypoth. 139·5575 miles.

6. In the triangle G H I, the baſe is 100, and the angle G 62° 27′: required the perpendicular and hypothenuſe?

Anſ. { Perp. 191·6896.
Hypoth. 216·2056.

7. What are the angles and baſe of that triangle, whoſe perpendicalar is 107·3, and hypothenuſe 220?

° ′ ″
Anſ. { 29 11 20 } The angles.
60 42 32 }
Baſe 192·0588.

8. On

8. In the triangle KLM, the baſe KM is 630, and the hypothenuſe KL 800, required the perpendicular and the angles?

Anſ. Perp. LM=493·051715.
° ′ ″
Angle K=48 2 52.
L=41 57 8.

9. In the triangle ABC, are given the perpendicular BC 35 miles, and the baſe CA 20 miles, to find the angles and hypothenuſe.

° ′ ″
Anſ. <A=60 15 19.
<B=29 44 41.
Hypoth. 40·28407 miles.

10. What are the angles and hpothenuſe of that triangle, whoſe baſe CA is 100, and perpendicular BC 90?

° ′ ″
Anſ. <A=41 59 14.
<B=48 00 46.
Hypoth. AB=134½.

11. Given the perpendicular 106 leagues, and the hypothenuſe 372 leagues, to find the baſe.

Anſ. 356·527698 leagues.

12. To find the angles and hypothenuſe of that triangle, whoſe baſe is 374 miles, and perpendicular 400 miles.

		°	′	″
Anſ.	Angle oppoſite the baſe	43	4	34.
	———————— perp.	46	55	26.
	Hypothenuſe	547·60933 miles.		

13. What will the angle at the baſe, and the hypothenuſe of that triangle be, whoſe perpendicular is 37, and baſe 64?

		°	′	″
Anſ.	Angle at the baſe	30	1	59.
	Hypothenuſe	73·925639.		

Reſolution

Reſolution of oblique angled Plane Triangles.

THEOREM III. *

FIG. 26.

The ſides of any triangle, whether it be right angled or obtuſe, are in direct proportion to the ſines of their oppoſite angles.

Note.

---

* To demonſtrate this Theorem, let C B E be an oblique angled triangle, produce C B, and on C E and C A let fall the perpendiculars B D and E A.

Then by Theorem 1. C B : R :: B D : S.C.
And B D : S.E :: B E : R.

By equality C B : S.E :: B E : S.C

Alſo,
As C E : R :: AE : S.C.
And A E : S.B :: B E : R.

By equality C E : S.B :: B E : S. C.
which is the ſame as the Theorem.

to Face page 50.

Fig. 1.

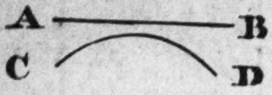


Fig. 2.

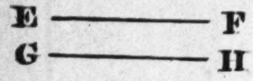


Fig. 3.

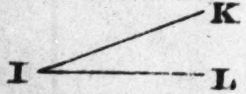


Fig. 4.

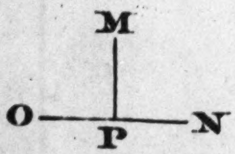


Fig. 5.

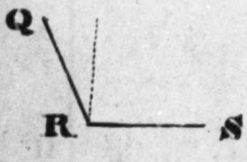
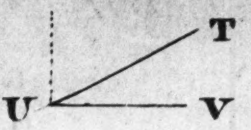


Fig. 6.



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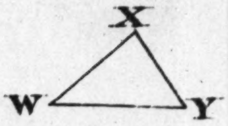

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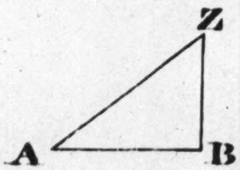

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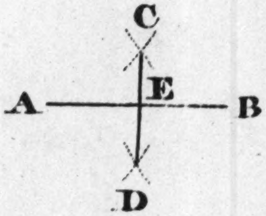


Fig. 10.

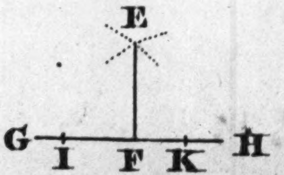
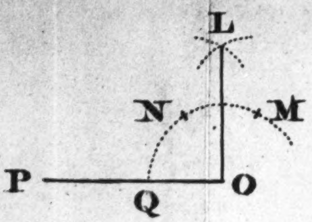


Fig. 11.



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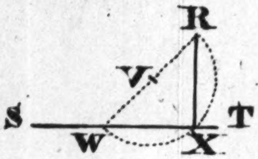


Fig. 13.

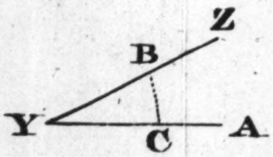


Fig. 14.

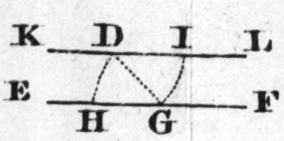


Fig. 15.

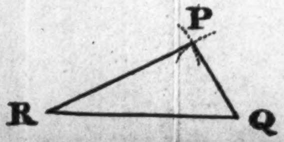


Fig. 16.

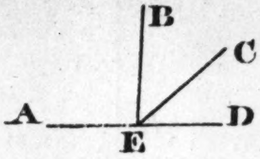


Fig. 17.

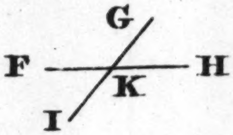


Fig. 18.

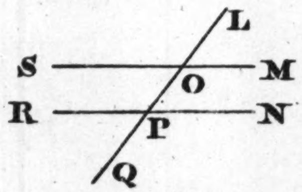


Fig. 19.

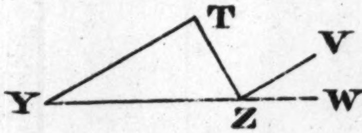
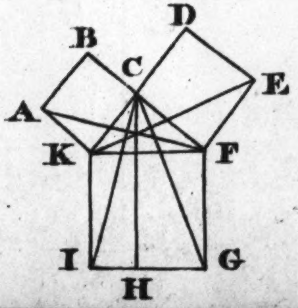


Fig. 20.



6.

20.

Fig. 21.

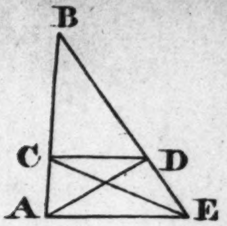


Fig. 22.

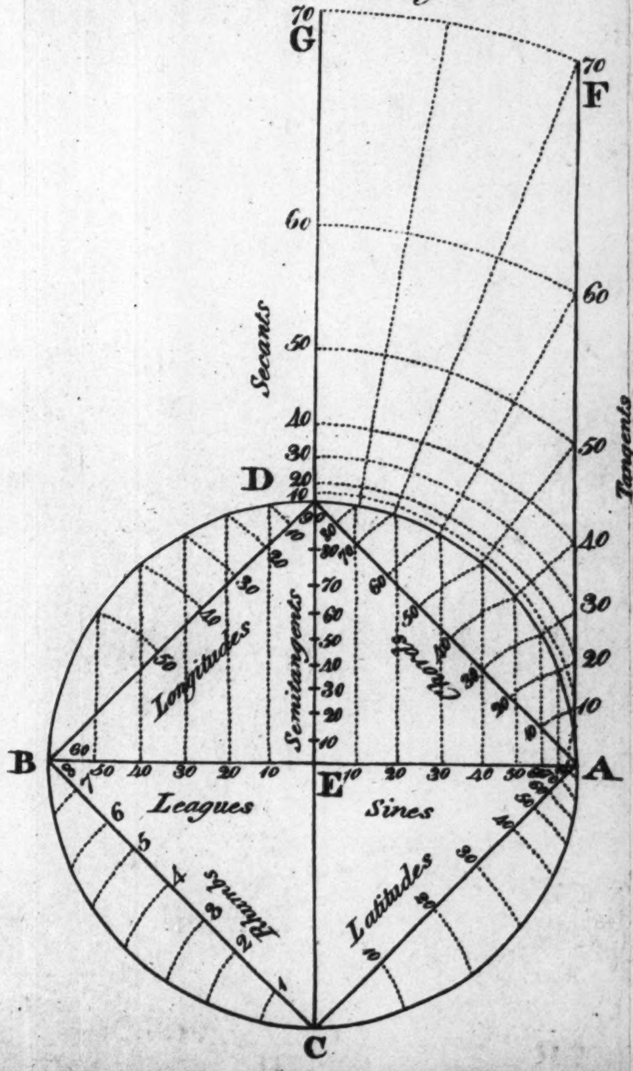


Fig. 23.

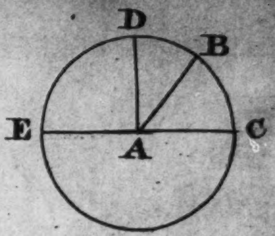


Fig. 24.

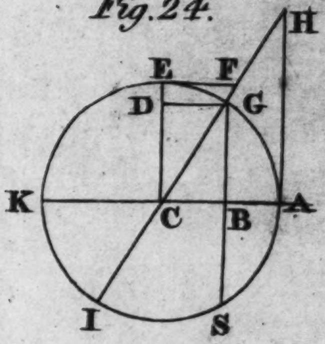


Fig. 25.

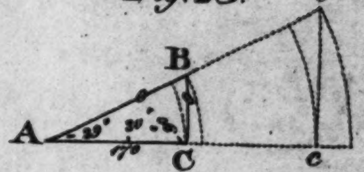
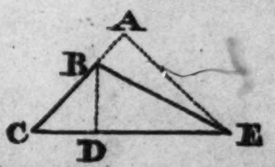


Fig. 26.



Note. Becauſe the ſine of an angle is alſo the ſine of any other angle which is the ſupplement of the former; therefore in this caſe the angle found by this Theorem is doubtful, unleſs the magnitude of the given angle remove this ambiguity. But it may be obſerved, that if the given angle be obtuſe, then the required angle will be acute; but when a given angle is acute, and oppoſite a leſſer ſide, then the angle ſought is ambiguous, and the geometrical conſtruction will produce two triangles; therefore every triangle in which there is contained an ambiguity, ought to have ſome reſtriction or limitation laid upon it before the operation.

EXAMPLES.

FIG 27.

1. In the oblique triangle A B C, there are given the angle A 39° 45′, B 85° 5′, and the ſide A C 35 leagues, to find the other two ſides.

GEOMETRICALLY.

1. Draw A C, and from a ſcale of equal parts, apply thereto 35 leagues.

2. Make the angle A equal to 39°, 45′, and the angle C (which is the ſupplement to the ſum of the angles A and B) equal to 55° 10′.

3. Draw A B and C B, and it is done.

Then A B and B C being meaſured by the ſcale of equal parts will be found to be 28·01 and 21·82 leagues.

ARITHMETICALLY.

The triangle ſimilar to the propoſed one, is the tabular triangle a b c, whoſe ſides by the table of natural ſines are as follow, viz. a b=820170, b c=6394390, and a c=9963204: hence by ſimilar triangles we have theſe analogies.

1. As { ac : cb :: AC : CB.
9963204 : 6394390 :: 35 : 21·82122 leagues.

2. As { ac : ab :: AC : AB.
9963204 : 8208170 :: 35 : 28·01085 leagues.

		Logarithms.
As ſi. angle B	85° 5′	9·9983990
Is to AC 35 leagues		1·5314789
So is ſi. angle A	39° 45′	9·8057991
To the ſide CB 21·82122 leagues		1·3388790

And,

As ſi. angle B	85° 5′	9·9983990
Is to AC	35 leagues	1·5314789
So is ſi. angle C	55° 10′	9·9144264
To the ſide AB 28·01085 leagues		1·4473263

INSTRUMENTALLY.

1. Extend from 85° 5′ to 39° 45′ upon the line of ſines, and that extent will reach from 35 to 21·82 upon the line of numbers.

2. Extend from 85° 5′ to 55° 10′ upon the line of ſines, then that extent will reach from 35 to 28·01 upon the line of numbers.

3. In the oblique triangle A B C, are given, the angle A 35° 18′, B 130° 17′, and the ſide A C 35, to find the other two ſides.

Anſ. Side A B 11·42163.
—— B C 26·51221.

4. In the triangle A B C, there are given A B 1360, B C 214, and the angle C 91° 30′, to find the other parts.

		°	′	″
Anſ.	Angle A -	9	2	27.
	—— B -	79	27	33.
	Side C A	1338·431.		

5. In the triangle A B C, are given A B 12 feet, A C 19 feet, and the angle B 90° 3″, to find the other ſide and angles.

		°	′	″
Anſ.	Side B C	14·93926 feet.		
	Angle A	51	50	17.
	—— C	38	6	43.

6. In the triangle A B C, are given A B 1000, the angle C 100°, and the angle B 30°, to find the two ſides.

Anſ. { Side B C 777·852.
—— C A 507·7132.

Two Sides and the Angle included by them being given, to find the other Parts of the Triangle.

THEOREM IV.*

FIG. 28.

As the ſum of the two given ſides :
Is to their difference - - ::
So is the tangent of half the ſum of the angles oppoſite theſe ſides, or cotangent of half the included angle - - - - :
To the tangent of half the difference of the two oppoſite angles.

D 2 Then

---

* Let A B C be an oblique angled triangle, and let B A, B C, and the angle B be the parts given.

Make

Then half the difference added to the half of the ſum of the two oppoſite

angles,

Make AE equal to BA, and draw BFG perpendicular thereto; alſo draw FD parallel to AC, then will $AF=FE$ and $ED=DC$. Alſo the angle $BAF=\frac{BAE+BEA}{2}=\frac{BAC+C}{2}$, and the angle $GAF=BAC-BAF=\frac{BAC-C}{2}$. Now the ſimilar triangles BFD, BGC give $BD:DC::BF:FG$. But $AF:R::BF:\text{tang.}\ BAF::FG:\text{tang.}\ FAG$, or $BF:FG::\text{tang.}\ BAF:\text{tang.}\ FAG$. Conſequently BA or $\frac{BE+BC}{2}:DC$ or $\frac{BC-BE}{2}::\text{tang.}\ BAF:\text{tang.}\ FAG$.

And that the half ſum of any two quantities, increaſed and diminiſhed by half their difference, gives the greater and leſs quantity reſpectively, may be proved thus :—Let the ſum of two given quantities $A+B=S$, and their difference $A-B=D$, then theſe two equations give $B=S-A=A-D$; and therefore $S+D=2A$. Conſequently the greater quantity $A=\frac{S+D}{2}$, and the leſs $B=\frac{S-D}{2}$.

angles, gives the greater angle; but if ſubtracted, the leſs.

So by theſe means all the angles become known, and the ſide required may be found by Theorem III.

EXAMPLES.

FIG 29.

1. In the plane triangle A B C, are given A B 36 poles, A C 48, and the angle A 42° 15′, to find the other parts.

GEOMETRICALLY.

1. From a ſcale of equal parts, lay off on A C 48.

2. Make the angle A equal to 42° 15′, by the line of chords.

3. Draw A B equal to 36, by the ſcale of equal parts.

4. Draw B C, and the triangle is geometrically conſtructed.

Then the required parts being meaſured by the proper ſcales, will be found to be as follow, viz. < B 89° 10′, < C 48° 35′ and the ſide B C $32\frac{1}{4}$ poles.

ARITHMETICALLY.

		Logarithms.
As the ſum of ſides AB+AC	84	1·9242793
Is to the diff. of ſides AC−AB	12	1·0791812
So is the tang. $\frac{<B+<C}{2}$	68° $52\frac{1}{2}$′	10·4129974
To the tang. $\frac{<B-<C}{2}$	20° 17′ 29″	9·5678993
Their ſum	89 9 59=angle B.	
Their diff.	48 35 1= —— C.	

Then,

As ſi. angle B	89° 9′ 59″	9·9999531
Is to the ſide A C	48 poles	1·6812412
So is ſi. angle A	42° 15′ 00″	9·8276063
To the ſide B C	32·27703 poles	1·5088944

INSTRU-

INSTRUMENTALLY.

1. Extend from 84 to 12 upon the line of numbers; then that extent will reach upon the line of tangents, from 68° 52½′ (the contrary way, becauſe the line of tangent is numbered back again from 45°) a little beyond 45°, which being ſet back ſo far from 45°, falls upon 20⅓′ for the fourth term in the firſt proportion.

2. Extend from 89⅙° to 42¼° upon the line of ſines, and that extent will reach from 48 to 32¼ upon the line of numbers.

2. In the triangle A B C, there are given A C 35, A B 28·01085, and the included angle A 39° 45′, to find the other parts.

$$\text{\textit{Anſ.}}\begin{cases} & \;\;\circ \quad\;\; ' \\ \angle B & 85 \quad 5. \\ \angle C & 55 \quad 10. \\ C\,B & 21{\cdot}82122. \end{cases}$$

3. In the triangle A B C are given B A 1360, B C 214, and the included angle B 79° 27′ 33″, to find the other parts.

		°	′	″
Anſ	<A	9	2	27
	<C	91	30	00
	CA 1338·431.			

The three ſides being given to find the Angles.

In any plane triangle, if a perpendicular be demitted from the greateſt angle upon the oppoſite ſide or baſe, it will divide it into two ſegments, and the whole triangle into two right angled triangles. Then,

THEO-

THEOREM V.*

FIG. 30.

As the baſe, or longeſt ſide :
Is to the ſum of the other two ſides ::
So is the difference of thoſe ſides :
To the difference of the ſegments of the baſe.

---

* Let A B C be the given triangle, and B E the perpendicular; from B as a centre, with the radius B C deſcribe a circle, cutting the baſe A C in E, and the ſide A B produced in D; then will A D be the ſum of the ſides A B and B C, and A G their difference; alſo A F the difference of the ſegments of the baſe A E and E C. Then (by *Eucl.* 36 III.) $AD \times AG = AC \times AF$, or $AC : AD :: AG : AF$, that is $AC : AB + BC :: AB - BC : AC - EC$.

And that the half ſum of two quantities, increaſed and diminiſhed by half their difference, gives the greater and leſs quantities reſpectively, we have already proved.

Then half this difference being added to, and ſubtracted from half the baſe, will give the greater and leſs ſegment reſpectively. Hence, in each of the right-angled triangles, there will be known two ſides, and the right-angle oppoſite one of them, and therefore the angles will be found by Theor. I.

Note. In the above analogy, if for the third term half the difference of the ſides be taken, then half the difference of the ſegments will be the fourth term.

EXAMPLES.

FIG. 31.

1. In the plane triangle A B C, are given A B 60, B C 50, and C A 80, to find the angles.

GEOMETRICALLY.

1. Draw A C equal to 80 by a ſcale of equal parts.

2. With the centres A and C, and radii 60 and 50, taken from the above ſcale, deſcribe arches interſecting in B.

3. Draw A B, B C and the triangle is completed.

Then the angles being meaſured, will be found to be as follow, viz.

$<A=38° 36' 30''$, $<B=92° 46'$, and $<C=48° 37' 30''$.

ARITHMETICALLY.

Having demitted the perpendicular B D we ſhall have,

As $AC=80 : AB+BC=110 :: AB-BC=10 : AD-DC=\frac{110\times10}{80}=13{\cdot}75$

Hence, $\frac{13{\cdot}75+80}{2}=46{\cdot}875=AD$,

And, $\frac{80-13{\cdot}75}{2}=33{\cdot}125=DC$.

Then in the right angled triangle A D B, by Theor. I. it will be, As 60 (B A) : rad. 1 (ſi. <A D B=90°) :: 46·875 (A D) : 7814299 (nat. ſi. <B=51° 23′ 30″) which taken from 90° leaves 38° 36′ 30″ = <A.

Again in the triangle B D C,

As 50 (B C) : rad. 1 (nat. ſi. <D=90°) :: 33·125 (D C) : 6609763 (nat. ſi. <B=41° 22′ 28″) which taken from 90°, leaves 48° 37′ 32″= <C.

	°	′	″	
Hence, to	51	23	30	<A B D,
add	41	22	28	<D B C
The ſum is	92	45	58=	<A B C.

BY THE LOGARITHMS.

After the perpendicular B D is let fall, and A D, D C found as above, ſay,

		°	′	″	
As A B - - - -		60	—	—	1·7781513
Is to ſi. <D - -		90	00	00	10·0000000
So is A D -	46·875		-	-	1·6709413
To ſi. <A B D -		51	23	30 -	9·8928900
Which taken from		90	00	00	
leaves		38	36	30=	<A.

Again, in the triangle B C D,

		°	′	″	
As B C	- - -	50	—	—	1·6980700
Is to ſi. <D	- -	90	00	00	10·0000000
So is D C	- - 33·125		-		1·5191559
To ſi. <D B C	-	41	22	28	9·8201859
Which taken from		90	00	00	
leaves		48	37	32=<C	

And the angle A B C may be found as above by taking the ſum of the angles A and C.

INSTRUMENTALLY.

1. Extend from 80 to 110 upon the line of numbers, and that extent will reach upon the ſame line, from 10 to $13\frac{3}{4}$, the difference of the ſegments of the baſe.

2. Extend from 60 to $46\frac{7}{8}$ upon the line of numbers, and that extent will reach, upon the line of ſines, from 90° to $51\frac{1}{3}$.

3. Extend from 50 to $33\frac{1}{7}$ upon the

line of numbers, that will reach from 90° to $41\frac{1}{3}'$, upon the line of ſines.

2. In the triangle A B C, there are given A B 81, A C 135, and B C 108, to find the angles.

		°	′	″
Anſ.	<A	53	07	48.
	<B	90	00	00.
	<C	36	52	12.

3. In the triangle A B C, are given A B 22·53, A C 24, and B C 22·4, to find the angles.

		°	′	″
Anſ.	<A	57	27	00.
	<B	64	34	21.
	<C	57	58	39.

In the plane triangle A B C, there are given, A B 106, A C 106, and B C 184·72 to find the angles.

		°	′
Anſ.	<A	121	14
	<B	29	23
	<C	29	23

ALTI-

ALTIMETRY.

PART III.

The Application of Trigonometry to Altimetry and Longimetry.

ALTIMETRY is the art of meaſuring the *heights* of objects trigonometrically; and *Longimetry* is that of meaſuring their *diſtances*: and this in all caſes, whether they are or are not acceſſible.

Acceſſible diſtances are meaſured by applying to them ſome certain meaſure, as a pole, yard, foot, &c. ſome certain number of times.

Inacceſſible diſtances, are meaſured by means of angles, or by the properties of ſimilar triangles drawn from the principles of geometry: ſo when the angles are taken by ſome inſtrument, then the diſtances are determined by trigonometrical calculation; but when no angles are taken, then the unknown diſtances are calculated by the principles of ſimilar triangles, without paying any regard to the quantities of the angles.

The moſt uſeful inſtruments for taking of angles, are good quadrants, plain tables, or theodolites, &c.

To take an Angle of Elevation with the Quadrant.

FIG. 32.

Let A be any object, as ſuppoſe the moon, and let it be required to find the meaſure of the angle which it makes with the horizontal line C B.

Fix the centre of the quadrant at C, and move it round there, with one eye ſhut, till you ſee the moon at A, thro the two ſights E and D; then will the arch G H of the quadrant, cut off by the plumb C G, be the meaſure of the angle A C B.

To take an Angle of Depreſſion.

FIG. 33.

This is the ſame as the laſt, excepting that the eye muſt be applied to the centre, and the arch G H is the meaſure of the angle A C B.

To take an Horizontal Angle with the Protractor.

FIG. 34.

Let it be required to find the horizontal angle E C D, formed by the lines C E and C D, drawn from the two ſhrubs to the point C. Lay the edge C B of the protractor along the line C D, then the other line C E will cut the arch of the protractor in F, and conſequently the degrees, &c. contained in the arch F B, will be the meaſure of the required angle.

ALTIMETRY EXEMPLIFIED.

FIG. 35.

EXAMPLE I. Having meaſured A B equal to 150 feet, from the bottom of a tall fir-tree, in a right line upon a horizontal plane; I then with my quadrant took the angle of elevation of the top, and found it to be 39° 30′—Required the height of the tree, ſuppoſing the centre of the quadrant to be 5½ feet above the ground?

To find EB, by Theor. II. it will be,

As radius - - - - - -		10·0000000
Is to AB - -	150 -	2·1760913
So is tang. < A -	39° 30′	9·9161045
To EB - - -	123·6404	2·0921958
To which add AD=BC	5·5 - feet.	
	129·1404 feet=EC.	

FIG. 36.

2. Wanting to know the height of a tower, AB, which I could not approach, by reaſon of a river at ſome diſtance from the foot of it, I took an angle of elevation C=50° 25′, and going 130 yards farther from it, upon a horizontal plane, I found the angle of altitude there to be 36°—Required the height of the ſaid tower, and alſo

my diſtance from the bottom of it at my firſt ſtation?

	°	′	
<C	50	25	ſubtract.
<D	36	00	
	14	25	=<C A D.

Now in the oblique triangle A C D, the angles are known, and the ſide C D to find the ſide A C, by Theor. III. Part 2.

As ſi. <C A D	- -	14° 25′	9·3961499
Is to ſide C D	-	130 yards	2·1139434
So is ſi. <D	-	36° 00′	9·7692187
To ſide A C	- -	- -	2·4870122

Then in the right angled triangle A B C, by Theor. I. Prat 2.

As ſi. <B	-	90° 00′	10·- - - - -
Is to A C	- - -	-	2·4870122
So is ſi. <A C B	-	50° 25′ -	9·8868846
To the Tower's height AB		236·5356	2·3738966
And ſo is Co-ſi. <ACB		50° 25′ -	9·8042757
To the diſtance thereof BC		195·5635	2·2912879

Note. When an obſervation is taken by a quadrant, if the angle of altitude be 45°, then is the altitude of the object equal to the diſtance with the height of the centre of the inſtrument added.

FIG. 37.

3. To find the perpendicular height of a cloud whoſe angles of elevation are 30° 36′ and 63° 13′ taken by two obſervers, at the ſame time, both on the ſame ſide of the cloud, and at the diſtance of 1000 yards from each other; they being ſo placed that a vertical plane would paſs through both their ſtations and the cloud; and likewiſe to find its diſtance from each obſerver.

	°	′		°	′
< C -	90	00	< C -	90	00
< D -	63	13	< A -	30	36
< DBC	26	47	< ABC	59	24
			< DBC	26	47
			< ABD	32	37

Then

Then, as ſi. < A B D -	32° 37′	9·7316015
Is to A D - - -	1000	3· - - - - -
So is ſi. < A - -	30° 36′	9·7067531
So the diſtance B D	944·3904 yds.	2·9751516

Again, in the oblique triangle A B D.

As ſi. < A B D - - - -	32° 27′	9·7316015
Is to A D - - - - - - -	1000	3· - - - - - -
So is ſi. < A D B - - -	116° 47′	9·9507138
To diſtance A B	1656·2 yards	3·2191123

To find B C the height of the cloud.

As ſi. < C - - -	90° 00′	10·
Is to B D 944·3904 yards	- -	2·9751516
So is ſi. < B D C - -	63° 13′	9·9507138
To B C - - -	843·0736 yards.	2·9258654

FIG. 38.

4. It has been obſerved, that the ſteeple of *Boſſall church* in *Yorkſhire*, caſts a longer ſhadow on the 21ſt of December at noon, than on the 21ſt of June at noon, by 70 yards: required

the height of the ſaid ſteeple, ſuppoſing the ſun's altitude on the former of theſe days to be 13° 32′, and on the latter 59° 28′?

	°	′	
< BDC	59	28	ſubtracted.
< A	12	32	
< ABD	46	56	

As ſi. < ABD	46° 56′	9.8636557
Is to AD	70 yards	1·8450980
So is ſi. < ADB	120° 32′ or 59° 28′	9·9351715
To AB		1·9166138
And, as ſi. < C	90° 00′	10·-------
Is to AB		1·9166138
So ſi. < A	12° 32′	9·3364749
To the ſteeple's height BC 17·90792 yards		1·2530887

FIG. 39.

5. In Gilling Park there is a ſummer-houſe, ſtanding upon the top of a declivity, from the bottom of which I meaſured a diſtance of 35 feet, and then

with a quadrant took the angle formed by the plane, and a line drawn to the top 39° 30′, and proceeding on in the ſame direction 53 feet farther, I found the angle to be 22° 25′, the height of the quadrant at both obſervations was 6 feet: The height of the building is required?

	°	′	
< D	39	30	ſubtracted.
< E	22	25	

Then, as ſi. < ECD	17 05	9·4679960
Is to DE - -	53 feet	1·7242759
So is ſi. < E	22° 25′	9·5813116
To CD 68.8	feet	1·8375915

Then, as BD+DC=103·8		2·0161974
Is to DC−BD=33·8		1·5289167
So is tang. $\frac{B+BCD}{2}$	=70° 15′	10·4448612
To tang. $\frac{B-BCD}{2}$	=42 12	9·9575805
Diff.=<BCD	28 3	

Laſtly, to find C B, ſay,

As ſi. < B C D - - - -	28° 3′	9·6723213
Is to B D - - - - - -	35 feet	1·5440680
So is ſi. < C D B - -	39° 30′	9·8035105
To C B - -	47·34315 feet -	1·6752572
Add E F	6·	the quadrant's height.
C A	53·34315	the whole height of the building.

Beſides the methods above exemplified, there are various others which in many caſes are more ſimple.

FIG. 40.

1. Suppoſe it was required to find the height of the maypole A B.

Set up a ſtaff a b at b, and meaſure its ſhadow c b, ſuppoſe 10 feet, as alſo the ſhadow b B of the maypole 22, and ſuppoſing the ſtaff a b = 11 feet; then the ſimilar triangles c a b and b A B give

$$\begin{cases} cb : ab :: bB : AB \\ 10 : 11 :: 22 : 24{\cdot}2 \text{ feet.} \end{cases}$$

2. Another method is by ſetting up two rods parallel to the object to be meaſured, the uttermoſt one longer than the other, ſo that the obſerver may ſee the top of the object over the tops of both the rods.

FIG. 41.

Suppoſe the rod f g be $=2$ feet, c e $=3$ feet, their diſtance aſunder g c or f d $=4$ feet, and the diſtance g B of the ſhorter rod from the object $=82$ feet. Then the ſimilar triangles f e d and f A C give

$$\text{As}\left\{\begin{array}{l} fd : de :: fC : CA \\ 4 : 3-2 :: 82 : 20{\cdot}5 \end{array}\right.$$

Hence $AC+fg=AC+CB=20{\cdot}5+2=22{\cdot}5=AB$.

FIG. 42.

3. Another method is by means of optical reflection.

Let a veſſel of water, or mirror, be placed horizontally at C, at the diſtance of 40 feet from the pole A B, and let a perſon at D, 4 feet from C, with his eye 5½ feet from the ground, view the image of the object at C. Then by by ſimilar triangles,

$$\text{As}\left\{\begin{array}{ccccccc} CD & : & DE & :: & CB & : & BA \\ 4 & : & 5\tfrac{1}{2} & :: & 40 & : & 55 \text{ feet.} \end{array}\right.$$

FIG. 43.

4. A fourth method is for the obſerver to fix a rod E C, at ſuch a diſtance from the foot of the object B A, that when he is laid upon his back, with his feet againſt the bottom of the rod, he may ſee the tops of the rod and object in the ſame line; then, if the diſtance from his eye to his foot be equal to the height of the rod E C, the diſtance of his eye from the bottom of the object will be equal to the object's height. Or, let the length of the rod be what it will, it will always hold, as D C : C E :: D B : B A; ſo if D C=5 feet, C E 4½ feet, and D B 15 feet; then will B A be 13½ feet.

FIG. 44.

5. The height of an object may be found, by knowing the utmoſt diſtance at which its top can juſt be ſeen in the horizon, by this proportion, viz. As 1 mile : $\frac{2}{3}$ of a foot :: the ſquare of any diſtance in miles : the height of the object in feet. And this rule is founded from the ſuppoſition, that the horizon dips 8 inches, or ^{2} of a foot, at the diſtance of one mile, and according to the ſquare of the diſtance for other diſtances.

FIG. 44.

Let it be required to find the height of the object A B, whoſe top can juſt be ſeen from C, whoſe diſtance C B is is 6 miles. Then by the analogy, we have, as 1 : $\frac{2}{3}$ :: 6^2, or 36 : 24 feet=AB.

LONGIMETRY EXEMPLIFIED.

FIG. 45.

Example i. Wanting to know the diſtannce between the two places A and B, to the ends of which there is free acceſs, but not to the intermediate parts, by reaſon of a hill between A and B; I therefore fixed my ſtation at C, and meaſured from thence to A, and found the diſtance to be 139 yards, and from C to B 185 yards, alſo the angle C I found to be 57° 30′: What is the diſtance between the two places?

As $CB+CA=343$ - - - - 2·5352941
Is to $CB-CA=46$ - - - - 1·6627578
So tang. $\frac{<A+<B}{2}$ = 61° 15′ 10·2607293
So tang. $\frac{<A-<B}{2}$ = 18 44 9·3881930

Sum 79 59=$<A$

Then, as ſi. $<A$ 79° 50′ - 9·9933292
Is to CB 185 yards - - - - 2·2671717
So ſi. $<C$ - - - 57° 30′ - 9·9260292
To the diſtance AB 165·96 yards 2·1998717

FIG. 46.

2. Being on the ſide of a river, and wanting to know the diſtance of a tree which ſtood on the oppoſite ſide, I meaſured A B 250 yards in a direct line by the ſide of the river, and found the two angles at each end of this line formed by the other end, and the tree to be 72° 35′ and 69° 30′: required the diſtance between each ſtation and the tree?

< A 72 35 } added.
< B 69 30 }

142 5 } ſubtracted.
180 00 }

< C 37 55

Then, as ſi. < C	37° 55′	-	9·7885323
To ſide A B	250 yards	-	2·5440680
So ſi. < B	69° 30′	-	9·9715876
To the ſide A C	533·5208	-	2·7271433
And, ſo is ſi. < A	72° 35′	-	9·9796182
To the ſide C B	543·4428	-	2·7351539

FIG. 47.

3. From a ſhip at ſea, I obſerved a point of land to bear E. by S. and after ſailing N. E. 20 miles, I ſet it again, and found its bearing to be S. E. by E. required my diſtance from the point of land at each obſervation?

By the table of rhumbs, the angle A, at the place of the firſt obſervation, is given 5 points, or 56° 15′, the angle B 9 points, or 101° 15′, and conſequently the angle C 2 points, or 22° 30′; and A B 20 miles, to find the ſides B C and C A. Hence,

As ſi. < C - - - -	22° 30′ -	9·5828397
Is to A B - -	20 miles - -	1·3010300
So is ſi. < A - -	56° 15′ - -	9·9198464
To B C -	43·4547 miles - -	1·6380367
And, ſo is ſi. < B	101° 15′ - -	9·9915739
To C A -	51·25829 miles	1·7097642

FIG. 48.

4. Wanting to know the diſtance between a houſe and a mill which were ſeparated from me by a river, I took another ſtation B, at the diſtance of 300 yards from the firſt A: now, from the firſt ſtation A, the angle ſubtended by B and the mill was 58° 20′, and by the mill and the houſe 37°; from B, the angle ſubtended by A and the houſe was 53° 30′, and by the houſe and the mill 45° 15′: What is the diſtance of the houſe and mill?

$$\begin{array}{l} \\ <HAM \\ <MAB \\ <HBA \end{array} \begin{array}{rr} ^{\circ} & ' \\ 37 & 00 \\ 58 & 20 \\ 53 & 30 \end{array} \!\!\!\!\begin{array}{l} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{added.} \end{array}$$

$$\begin{array}{rr} 148 & 50 \\ 180 & 00 \end{array} \Big\} \text{ſubt.}$$

$$\underline{\underline{31 \quad 10}} = <BHA.$$

$$\begin{array}{l} \\ <MAB \\ <HBM \\ <HBA \end{array} \begin{array}{rr} ^{\circ} & ' \\ 58 & 20 \\ 45 & 15 \\ 53 & 30 \end{array} \!\!\!\!\begin{array}{l} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{added.} \end{array}$$

$$\begin{array}{rr} 157 & 5 \\ 180 & 00 \end{array} \Big\} \text{ſubt.}$$

$$\underline{\underline{22 \quad 55}} = <BMA.$$

$$\begin{array}{l} \\ <HBM \\ <HBA \end{array} \begin{array}{rr} ^{\circ} & ' \\ 45 & 15 \\ 53 & 30 \end{array} \Big\} \text{added.}$$

$$98 \quad 45 = <MBA.$$

Now in the triangle A H B,

As ſi. < B H A - 31° 30′ - 9·7139349
Is to A B - - - - - 300 - - 2·4771213
So is ſi. < H B A - - 53° 30′ - 9·9051787

To H A 465·9776 - - - - - - 2·6683651

Again, in the triangle A M B,

As ſi. < B M A - - 22° 55′ - 9·5903869
Is to A B - - - - - 300 - 2·4771213
So ſi. < M B A - - 98° 45′ - 9·9949158

To A M 761·4655 - - - - - - 2·8816502
H A 465·9776

1227·4431 ſum.

295·4879 difference.

In the triangle A H M, we have

As A M+H A 1227·4431 - - 3·0890013
Is to A M−H A 295·4879 - - 2·4705397
So tang. $\frac{AHM+AMH}{2}$ 71° 30′ 10·4754801

To tang. $\frac{AHM-AMH}{2}$ 35 44 9·8570185

The diff. 35 46=<H M A.

Laſtly, in the triangle A H M,

As ſi. < H M A	35° 46′	9·7667739
Is to H A	465·9776	2·6683651
So ſi. < H A M	37° 00′	9·7794630
To the diſt. M H	479·7933 yards	2·6810542

FIG. 49.

5. The moon's horizontal paralax, at her mean diſtance from the earth, is 57′ 5″: It is required to find her diſtance from the earth's ſurface, ſuppoſing the earth's ſemidiameter to be $3928\frac{7}{8}$ miles?

As ſi. < ☽	57′ 5″	8·2202155
Is to A B	$3928\frac{7}{8}$ miles	3·5942655
So ſi. < A	90°	10·------
To ☽ B	236619 miles	5·3740500
B C	$3928\frac{7}{8}$ ſubtract.	
C ☽	$232690\frac{1}{8}$	

6. Let it be required to find the diameter of the moon in miles, when her diſtance from the earth is 236619 miles, and her apparent diameter in the heavens is 31′ 16″.

FIG. 50.

In the triangle E M C, we are preſented with E C the diſtance of the moon from the earth, 23619 miles, the angle E=15′ 38″, and the angle M=90°, to find M D.

	° ′ ″	
Hence, as ſi. < M	90 00 00	- 10·
Is to E C 236619	- - -	5·3740500
So ſi. < E -	0 15 38	- 7·6577762
To M C 1076·034	- -	3·0318262
Which mult. by	2	

Gives 2152·068 miles=M D.

FIG. 51.

7. How many miles can mount Etna, in Italy, be ſeen, whoſe height is $2\frac{1}{8}$ miles, ſuppoſing the earth to be a perfect ſphere, whoſe diameter is $7857\frac{3}{4}$ miles?

Firſt, draw the line C A, and the tangenial line A B; then will C E= $\frac{1}{2}$ D E=$3928\frac{7}{8}$ miles, to which add $2\frac{1}{8}$ miles (E A) gives C A=3931 miles.

Hence, as A C 3931 - -	3·5945030
Is to ſi. < B 90° -	10·------
So B C $3928\frac{7}{8}$ -	3·5942655
Si. < A 88° 6′ 19″	9·9997625
Then as radius - - - - -	10·-------
Is to B C $3928\frac{7}{8}$ miles -	3·5942655
So is tang. < C 1° 53′ 21″	8·5183051
To A B – 129·5897 miles -	2·1125706

Or, thus, by Cor. 1. to Theor. 5. Part 1.

$\sqrt{CA^2-CB^2}$=AB=129·5897 miles, the ſame as above.

FIG. 52.

8. Wanting to know my diſtance from an inacceſſible object, A, and having no inſtrument for taking angles, but only a chain for meaſuring diſtances; from each of the two ſtations B and C, which are 200 yards aſunder, I meaſured in a direct line from the object A 40 yards, viz. C D and B E each equal to 40 yards; the diagonal B D, I meaſured to 220 yards, and C E 230 yards—Required the diſtance of the object from each of the ſtations B and C?

Demit the perpendicular C F and B G; then in the triangle C B E,

$$\text{As}\left\{\begin{array}{l} CE : CB+BE :: CB-BE : CG-GE \\ 230 : 200+40 :: 200-40 : 166\tfrac{22}{23}. \end{array}\right.$$

$$\begin{array}{lr} \text{Half of which is} & 83\tfrac{11}{23} \\ \text{Half ſum} & 115 \\ \hline & 198\tfrac{11}{23}=CG \\ & 31\tfrac{12}{23}=GE \end{array}$$

Then, ° ′ ″

As CB 200 : 1 :: CG $198\frac{11}{23}$: ſ. <CBG 82 55 39

And,

As BE 40 : 1 :: GE $31\frac{12}{23}$: ſ. <GBE 52 00 11

The ſum 134 55 50

taken from 180 00 00

leaves 45 4 10

= <CBA

Again, in the triangle DCB,

$$\text{As}\left\{\begin{array}{l} DB : CB+CD :: CB-CD :: FB-DF \\ 220 : 200+40 :: 200-40 : 179\frac{1}{11} \end{array}\right.$$

its half $89\frac{6}{11}$

half DB 110

EB - - - $199\frac{6}{11}$

DF - - - $20\frac{5}{11}$

Then, ° ′ ″

As CB 200 : 1 :: FB $199\frac{6}{11}$: ſ. <FCB 86 8 10

And,

As DC 40 : 1 :: DF $20\frac{5}{11}$: ſ. <DCF 30 45 16

The ſum 116 53 26

taken from 180 00 00

leaves <ACB 63 6 34

add <CBA 45 4 10

the ſum 108 10 44

taken from 180 00 00

leaves the <A 71 49 16

Then,

	°	′	″	
As ſi. < A - - - - - - - -	71	49	16 -	9·9777633
Is to C B - - 200 yards - - - -				2·3010300
So is ſi. < C B A - -	45	4	10	9·8500107
To the diſt. C A 149·0312 yards -				2·1732774
And, So is ſi. < A C B	63	6	34	9·9503039
To the diſt. A B 187·7459 yards -				2·2735706

FIG. 53.

9. A ſhip ſailing N.W. two iſlands appear in ſight, of which one bore N. and the other N.N.W. but after ſailing 20 leagues, the former bore N.E. and the latter W. by S.—What is the diſtance aſunder of the two iſlands?

At the place of the firſt obſervation, there are given the angles B D C and C D A; the former of which is 2 points, or 22° 30′, and the latter, 4 points, or 45°; and at the ſecond place of obſervation are given the angle D C A, eight points, or 90°, and the angle D C B, 11

11 points, or 123° 45′; alſo DC 20 leagues, to find AB.

Becauſe the angle DCA is 90°, and CDA 45°, the angle CAB will be 45°, and conſequently CA=CD=20 leagues.

	°	′
<BDC	22	30
<DCD	123	45
The ſum	146	15
ſubtracted from	180	00
leaves - -	33	45=<CBD.

Then,

	°	′	
As ſi. <CBD - - -	33	45	9·7447390
Is to DC 20 leagues - - - -			1·3010300
So is ſi. <BDC -	22	30	9·5828397
To BC 13·77624 leagues - - -			1·1391307

	°	′
<BCD	123	45
<DCA	90	00
The ſum	213	45
ſubtracted from	360	00
leaves <BCA	146	15
which taken from	180	00
leaves - - -	33	45=<A+<B.

Hence, in the triangle A B C,

As B C + C A 33·77624 - - 1·5286113

Is to CA − B C 6·22376 - - 0·7940530

So is tang. $\frac{<B+<A}{2}$ 16° 52′ 30″ - 9·4819129

To tang. $\frac{<B-<A}{2}$ 3 11 56 - 8·7473546

< A 13 40 34 diff.

Again, in the ſame triangle,

As ſi. < A 13° 40′ 34″ - 9·3737080

Is to B C 13·77624 leagues - 1·1391307

So is ſi. < C 146° 15′ or 33° 45′ - 9·7447396

To the diſt. A B 32·37137 leagues 1·5101610

10. Two parts lie eaſt and weſt of each other; a ſhip ſails from each, viz. the ſhip from the weſt port ſails N. E. 89 leagues, and the other ſails 80 leagues, when ſhe meets the former—Required the latter ſhip's courſe, and the diſtance between the two ports?

FIG. 54.

Here are given the angle A 4 points, or 45°, the ſide A B 89 leagues, and

B C 80 leagues, to find the angle C, the latter ſhip's courſe, and A C the diſtance between the two ports.

Hence,

As the ſide B C 80 leagues		- -	1·9030900
Is to ſi. < A	45° 00′	- -	9·8494850
So is A B 89 leagues		- -	1·9493900
To ſi. < C	51° 52′	- -	9·8957850

	°	′
< A	45	00
< C	51	52
The ſum	96	52
taken from	180	00
leaves	83	08=< B.

Then, as ſi. < A - -	45° 00′ -	9·8494850
Is to B C 80 leagues	- - -	1·9030900
So is ſi. < B -	83° 8′ -	9·9968736
To A C 112·35 leagues	- -	1·0505786

PRO-

PROMISCUOUS QUESTIONS.

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1. AT 85 feet diſtance from the bottom of a maypole, the angle of elevation was 52½—Required the height of the pole?

Anſ. 110½ feet.

2. In a well are given, the breadth of the top 6 feet, and the angle formed by its ſide, and a viſual diagonal line from the edge at the top to the oppoſite ſide at the bottom, 18° 30′, to find its depth?

Anſ. 17·89 feet.

3. From the top of a ſhip's maſt, which was 80 feet above the water, the angle of depreſſion of another ſhip's hull upon the water, at a diſtance was 20°—Required their diſtance?

Anſ. 219·79 feet.

4. Wanting

to Face page 96.

Fig: 27.

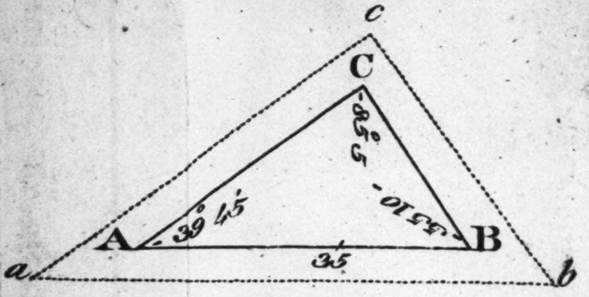


Fig. 28.

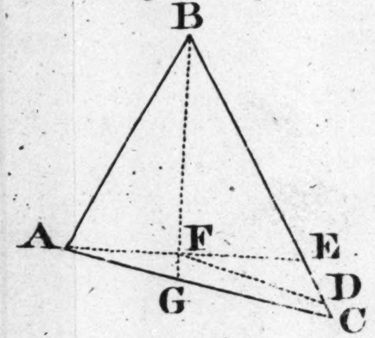


Fig. 29.

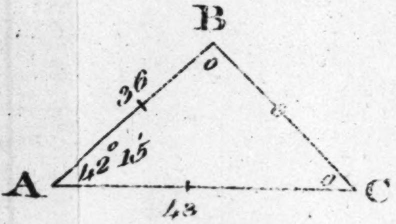


Fig. 30.

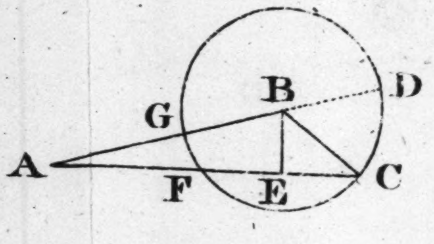


Fig. 31.

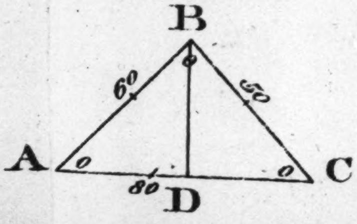


Fig. 32.

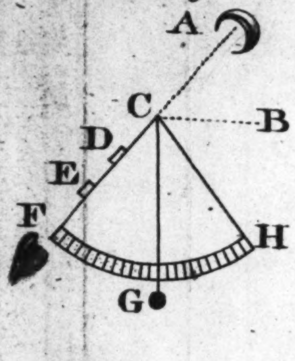
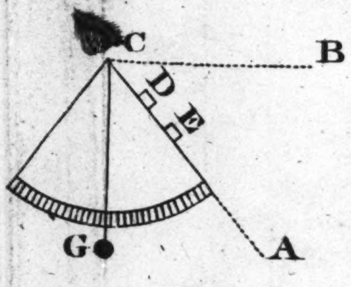


Fig. 33.



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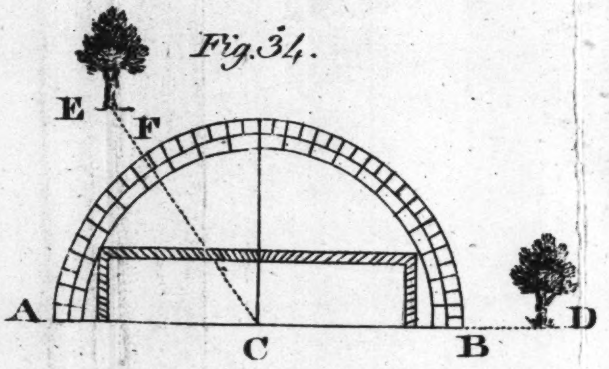

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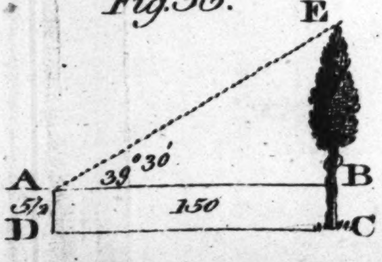

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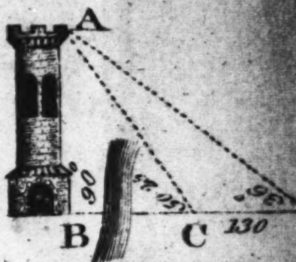
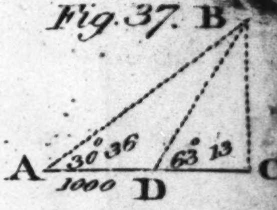


Fig. 37.



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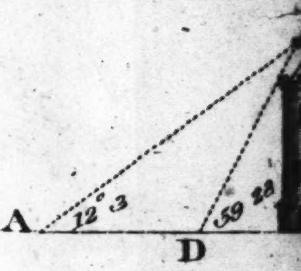
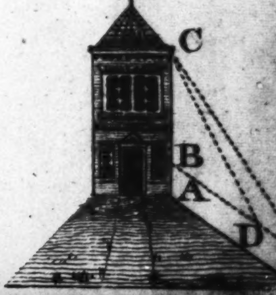


Fig. 39.



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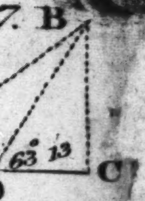

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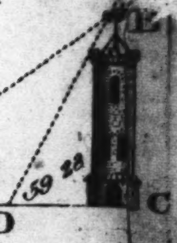

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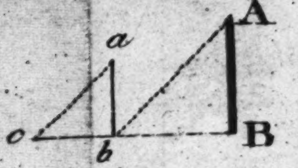
Fig. 40.



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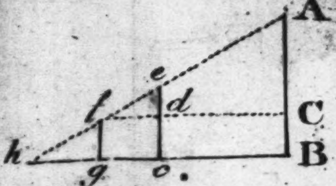

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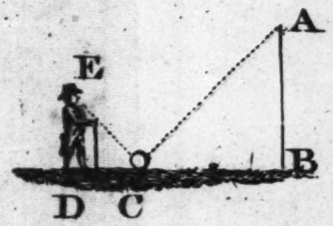
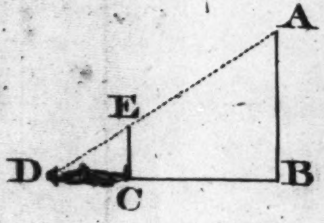


Fig. 43.



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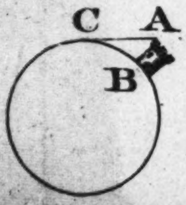

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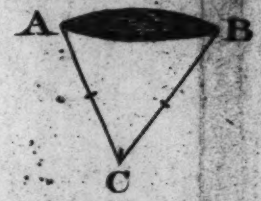


Fig. 46.

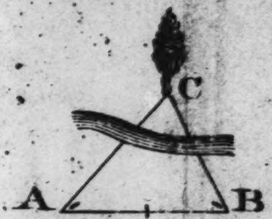


Fig. 47.

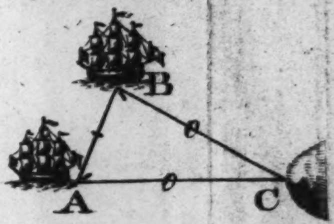


Fig. 48.

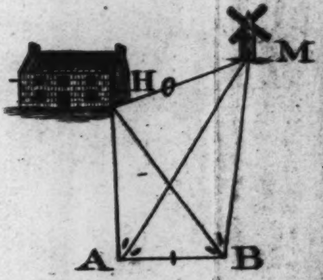


Fig. 49.

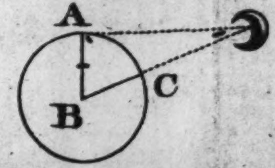


Fig. 50.

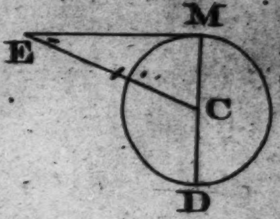


Fig. 51.

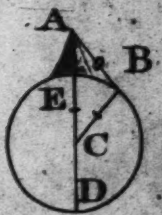


Fig. 52.

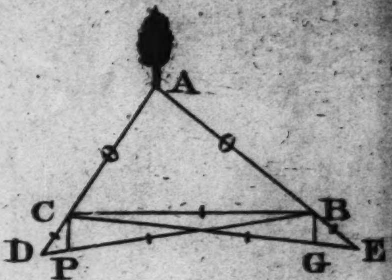


Fig. 53.

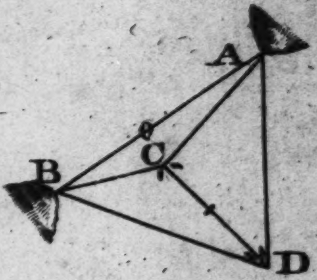
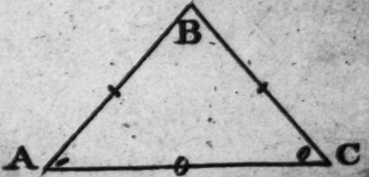


Fig. 54.



4. Wanting to know the height of an inacceſſible object, at the leaſt diſtance from it, upon the ſame horizontal plane, I took an angle of elevation equal to 58°, and going 100 yards directly farther from it, found the angle there to be only 32°—Required its altitude, and my diſtance from it, at the firſt ſtation, the inſtrument being 5 feet above the ground at each obſervation?

Anſ. { Height, 104·17 yds.
Diſt. 64·05 —

5. Wanting to know the height of, and my diſtance from, an object on the other ſide of a river, which ſeemed to be upon a level with the place where I ſtood by the ſide of the river, and not having room to go backwards, on the ſame plane, on account of the immediate riſing of the bank, I placed a mark where I ſtood, and meaſured in a direct line from the object up the

hill, whoſe aſcent was ſo regular that I might account it for a right line, to the diſtance of 132 yards, where I perceived that I was above the level of the object's top; I there took the angle of depreſſion of the mark by the river's ſide equal to 42°, of the bottom of the object equal to 27°, and of its top 19°—What is the height of the object, and the diſtance of the mark from the bottom?

Anſ. { Height 28·63 yards.
Diſtance 75¼ ——

6. Two ſhips ſet ſail from the ſame port, one of them goes 100 leagues due eaſt, and the other 168 leagues due ſouth—How far are they then aſunder?

Anſ. 195½ leagues.

7. A cord of 26¼ feet in length, will reach from the top of a wall 17 feet high, to the oppoſite ſide of a brook, which ſurrounds the wall—Required the breadth of the brook?

Anſ. 20 feet, nearly.

8. At a certain place, the angle of elevation of an inacceſſible tower was 26° 30′; then meaſuring 75 feet in a direct line towards it, the angle was found to be 51° 30′—Required the height of the tower, and its diſtance from the laſt ſtation?

Anſ. { Height 62 feet.
Diſtance 49 ——

9. A maypole whoſe top was broken off by a blaſt of wind, ſtruck the ground at 30 feet diſtance from the foot of the pole—What was the length of the whole maypole, ſuppoſing the length of the broken piece to be 63 feet?

Anſ. 118·39 feet.

10. If a ſhip in the latitude of 50° north, ſail 52 miles in the direction S.W. by S. what latitude is ſhe in, and how much farther to the weſtward?

Anſ. { Latitude 49° 16·8′
Weſt 28·9 miles.

11. Two ſhips ſail from the ſame port; the one ſails E.N.E. 85 miles, the other ſails eaſt by ſouth till the firſt ſhip bears N.W. by W.—What is the diſtance of the ſecond ſhip from the port, and alſo from the firſt ſhip?

Anſ. From the port 184·7
From the 1ſt ſhip 123·4

12. Two ſhips ſail from a certain port; the one ſails S. by E. 45 leagues, and the other S.S.W. 64 leagues—What then are their bearings and diſtance aſunder?

Anſ. Bearing 43° 14′
Diſt. 36·5 leagues.

13. To find the diſtance of an inacceſſible caſtle-gate, I meaſured a line of 73 yards, and at each end of it took the angle of poſition of the object and the other end, and found the one to be 90°, and the other 61° 45′—Required the diſtance of the gate from each ſtation?

Anſ. 135·8 yards.
154·2 ——

14. Two ſhips of war intending to cannonade a fort, are, by the ſhallowneſs of the water, kept ſo far from it, that they ſuſpect their guns cannot reach it; in order, therefore, to meaſure the diſtance, they ſeparate from each other half a mile or 880 yards; then each ſhip obſerves the angles which the other and the fort ſubtend, and finds them to be 85° 15′ and 83° 45′—What is the diſtance between each ſhip and the fort?

Anſ. 4584·5 and 4596·1 yards.

15. What is the perpendicular height of a cloud, whoſe angles of elevation are 35° and 64°, taken by two obſervers, at the ſame time, both on the ſame ſide of the cloud, and at the diſtance of 880 yards from one another, ſo placed that a vertical plane would paſs thro' both their ſtations and the cloud?

Anſ. 935·757 yards.

16. Wanting

16. Wanting to know the breadth of a river, I meaſured 100 yards in a ſtraight line cloſe by one ſide of it; and at each end of this line I found the angles ſubtended by the other end, and a tree, cloſe by the other ſide of the river, to be 53° and 79° 12′—What is the perpendicular breadth?

Anſ. 105·89 yards.

17. A maypole at a certain time caſts a ſhadow 106 feet long; and at the ſame time a ſteeple, 70 feet high, caſts its ſhadow 60½ feet—What is the height of the maypole?

Anſ. 91$\frac{23}{70}$ feet.

18. The height of an elm growing in the centre of a circular iſland, 30 feet in diameter, plumbs 53 feet; and a line ſtretched from the top of the tree, ſtraight to the nearer edge of the water is 112 feet—What is the

breadth of the moat, ſuppoſing the land on each ſide of the water to be level?

Anſ. 83⅔ feet.

19. From the top of a tower by the ſea-ſide of 143 feet high, I obſerved that the angle of depreſſion of a ſhip's bottom, then at anchor, was 55°—Required the ſhip's diſtance from the bottom of the tower?

Anſ. 204·56 feet.

20. From a window near the bottom of a houſe, which appeared to be upon a level with the bottom of a ſteeple, I took the angle of elevation of the top of the ſteeple equal to 40°, and from another window 18 feet directly above the former, the ſame angle was 37½°—Required the height and diſtance of the ſteeple?

Anſ. { Height 210·44 feet.
Diſtance 250·79 ——

21. There

21. There are two columns left ſtanding upright in the ruins of Perſepolis; the one is 64 feet above the plane, and the other 50; in a right line between theſe ſtands an ancient ſtatue, the head of which is 97 feet from the top of the higher, and 86 from that of the lower column; the baſe of which meaſures juſt 76 feet to the centre of the figure's baſe—What is the diſtance between the tops of the two columns?

Anſ. 157 feet, nearly.

22. There is a triangular field, whoſe three ſides are 6, 10, and 12 chains, reſpectively—What are the angles?

Anſ. 29° 56′, 93° 49′, and 56° 15′.

23. According to aſtronomers, the diameter of the ſun, at a mean diſtance from the earth, ſubtends an angle of 32′ 15″—Required how many times his

diameter in length is equal to his mean diſtance from the earth?

Anſ. 106·6189 times his diameter.

24. Obſerving three ſteeples, A, B, and C, in a town at a diſtance, whoſe diſtances aſunder are known to be as follow, viz. AB 106·5, AC 202, and BC 131 fathoms, I took their angles of poſition from the place where I ſtood, D, which was neareſt the ſteeple B, and found the angle ADB 13° 30′, and the angle CDB 29° 50′—Required my diſtance from each of the three ſteeples?

Anſ. $\left\{\begin{array}{l}\text{DA } 302{\cdot}8\\ \text{DB } 214{\cdot}8\\ \text{DC } 262{\cdot}0\end{array}\right\}$ fathoms.

25. Suppoſing my ſtation to be fartheſt from the ſteeple B—Required to find the diſtances from it, when the diſtance AB is 9 furlongs, AC 12, and BC 6

6 furlongs; alſo the angle A D B is 33° 45′, and C D B 22° 30′?

Anſ. DA 10·64, DB 15·64, DC 14·01 furlongs.

26. From the top of a tower, whoſe height was 120 feet, I took the angle of depreſſion of two trees which ſtood in a direct line, upon the ſame horizontal plane, with the bottom of the tower, namely, that of the nearer 57°, and that of the farther 25° 30′—Required the diſtance between the two trees, and the diſtance of the nearer from the bottom of the tower?

Anſ. Diſtance between the trees 173·656 feet.
Diſt. of the nearer from the tower's bottom 77·929 ft.

27. Being at York, and wanting to know the height of the Cathedral; with my quadrant I took the angle of elevation of its top equal to 21° 1′, and then going 116 yards in a direct line towards it, the angle of elevation was there

39°

39° 42′—Required the height of that noble ſtructure, and my diſtance from it at each obſervation, the quadrant being 6 feet from the ground?

Anſ. Height - - - - - 83 yards.
Diſtance at the firſt ſtation 216 ——
Ditto, at the ſecond ſtation 100 ——

28. To what diſtance muſt a perſon be raiſed from the centre of a ſphere, whoſe diameter is 7969·16 miles, to make it ſubtend an angle with his eye of $32\frac{1}{6}'$?

Anſ. 851802 miles.

29. How far at ſea can a mountain be ſeen, whoſe height is 300 yards, ſuppoſing the radius of the earth to be $3978\frac{7}{8}$ miles?

Anſ. $36\frac{1}{4}$ miles.

30. Required the height of a tower, whoſe top can juſt be ſeen in the horizon, at the diſtance of 19·326 miles?

Anſ. 249 feet.

31. Suppoſe a light-houſe, built on the top of a rock, the diſtance between

the place of obſervation, and the top of the rock level with the eye, and directly under the building, is given 310 fathoms; the diſtance from the top of the rock to the place of obſervation, is 423 fathoms, and from the top of the building 425 fathoms—What is the height of the light-houſe?

Anſ. 17·6 feet.

32. At what diſtance from the foot of a tower, whoſe height is 276 feet, muſt I be, to make the angle of elevation of its top equal to 53° 7′ 48″, ſuppoſing the inſtrument to be 6 feet above the ground?

Anſ. 216 feet.

33. At the end of a ſhip's courſe, her difference of latitude was found to be 545·9 miles, and her departure 314·5—What were her bearing and diſtance?

Anſ. { Bearing 29° 57′.
Diſtance 630 miles.

34. Going from York to Malton, in a direction of 42° to the northward of the eaſt, at 4 miles diſtance from York,

I took an obſervation at Sheriff-Hutton Caſtle, and found its bearing to be 7°, 30′ to the weſtward of the north; then going 7 miles on the road, I took another obſervation, and there found its bearing to be 3° 30′ to the northward of the weſt—Required my diſtance from the Caſtle at each of thoſe ſtations; my diſtance from York when I was at the leaſt diſtance from the Caſtle, or paſſing by it; and alſo my diſtance from it at York?

		Miles.
Anſ.	Diſtance at the firſt ſtation - - -	5·0862
	Diſtance at the ſecond ſtation - -	5·8768
	Diſt. from York when I paſſed the Caſtle	6·8809
	Diſtance from the Caſtle at York -	8·0568

35. Upon a horizontal plane I ſaw a tall pine-tree which ſtood perpendicular to the horizon, and due north of me; but after going N. W. 200 feet, it bore N. E. by E.; I there with my quadrant took the angle of elevation of its top equal to 31°—Required the height of the tree, the quadrant being 6 feet above the horizon?

Anſ. 104·19808 feet.

TABLE OF RHUMBS.

North.		Pts.	Qrs.	°	′	″	Pts.	Qrs.	South.	
		0	1	2	48	45	0	1		
		0	2	5	37	30	0	2		
		0	3	8	26	15	0	3		
N.by E.	N.by W.	1	0	11	15	00	1	0	S.by E.	S.by W.
		1	1	14	3	45	1	1		
		1	2	16	52	30	1	2		
		1	3	19	41	15	1	3		
N.N.E.	N.N.W.	2	0	22	30	00	2	0	S.S.E.	S.S.W.
		2	1	25	18	45	2	1		
		2	2	28	7	30	2	2		
		2	3	30	56	15	2	3		
N.E.by N	N.W.by N.	3	0	33	45	00	3	0	S.E.by S	S.W.by S.
		3	1	36	33	45	3	1		
		3	2	39	22	30	3	2		
		3	3	42	11	15	3	3		
N.E.	N.W.	4	0	45	00	00	4	0	S.E.	S.W.
		4	1	47	48	45	4	1		
		4	2	50	37	30	4	2		
		4	3	53	26	15	4	3		
N.E.by E	N.W.by W.	5	0	56	15	00	5	0	S.E.by E.	S.W.by W.
		5	1	59	3	45	5	1		
		5	2	61	52	30	5	2		
		5	3	64	41	15	5	3		
N.N.E.	W.N.W.	6	0	67	30	00	6	0	E.S.E.	W.S.W.
		6	1	70	18	45	6	1		
		6	2	73	7	30	6	2		
		6	3	75	56	15	6	3		
E.by N.	W.by N.	7	0	78	45	00	7	0	E.by S.	W.by S.
		7	1	81	33	45	7	1		
		7	2	84	22	30	7	2		
		7	3	87	11	15	7	3		
EAST.	WEST.	8	0	90	00	00	8	0	EAST.	WEST.

A Table of Engliſh Lineal Meaſures.

Inches.	Feet.	Yards.	Poles.	Chains.	Furlongs.	Mile.
12	1					
36	3	1				
198	16½	5½	1			
792	66	22	4	1		
7920	660	220	40	10	1	
63360	5280	1760	320	80	8	1

NOTE,

Three barley-corns = 1 inch.
Four inches = 1 hand.
Six feet, or 2 yards = 1 fathom.
Three miles = 1 league.
Sixty geographical, or 69½ ſtatute miles = 1 degree.

A TABLE ſhowing the Height of the Apparent Level above the True, for every Hundred Yards Diſtance upon the Earth's Surface, on the one Hand; and for every Mile, on the other.

Diſt.	Diff. of Level.	Diſt.	Diff. of Level.	
Yards.	Inches.	Miles.	Feet.	Inches.
50	$\frac{50}{89928}$	1	0	8
100	0·026	2	2	8
200	0·103	3	6	0
300	0·231	4	10	7
400	0·411	5	16	7
500	0·643	6	23	11
600	0·925	7	32	6
700	1·260	8	42	6
800	1·645	9	53	9
900	2·081	10	66	4
1000	2·570	11	80	3
1100	3·110	12	95	7
1200	3·701	13	112	2
1300	4·344	14	130	1
1400	5·038	15	150	0
1500	5·784	16	170	8
1600	6·580	17	192	8
1700	7·425	18	216	0
1760	8·000	19	240	8
		20	266	8

The Application of the above Table to ſeveral uſeful Purpoſes.

1. To find the height of the apparent level above the true, at any diſtance.—If the given diſtance be within the limits of the table, the correction of level will be found on the ſame line with it: Thus, at the diſtance of 900 yards, the correction is 2·081, or two inches nearly; and at the diſtance of 8 miles, the correction is 42½ feet. But if the exact diſtance be not found in the table, then multiply the ſquare of the given diſtance in yards, by 2·57, and cut off 6 places on the right for decimals, and the reſt are inches; or multiply the ſquare of the diſtance in miles by 66⅓ feet, and cut off 2 places on the right for decimals.

2. To find the extent of the viſible horizon, or how far can be ſeen from any given height, on a horizontal plane, as upon the ſea, &c. Suppoſe the eye of an obſerver, on the top of a tower, be at the height of 80¼ feet above the

horizon, he will then ſee 11 miles all round. Or from the top of a ſhip's maſt, the height of which is 112 feet, a perſon may ſee to the diſtance of about 13 miles. Alſo, when the top of a mountain, tower, or light in a light houſe, whoſe height is 112 feet, comes firſt into view of the eye on board a ſhip at ſea, the table ſhows that the diſtance of the ſhip from it is 13 miles, if the eye be at the ſurface of the water; but if the height of the eye on the ſhip be 66⅓ feet, then the diſtance will be increaſed by 10 miles, making the diſtance in all 23 miles.

3. Suppoſe a ſpring to be on one ſide of a hill, and a houſe on another hill oppoſite the former, with a valley between them, and that the ſpring ſeen from the houſe appears, by a levelling inſtrument, to be on a level with the foundation of the houſe, and let the diſtance of the ſpring from the houſe be 2 miles; then, by the table, the ſpring is 2 feet 8 inches, or 2⅔ feet above

the level of the foundation of the houſe, and ſo much fall would the water have, if brought in pipes from the ſpring to the houſe, the pipes being laid all the way in the ground. And by theſe means we can almoſt level to any diſtance at one operation, which the ancients could not do but by a great multitude; for, not being acquainted with the correction anſwering to any diſtance, they levelled from one 20 yards to another only, when they had occaſion to continue the work to ſome conſiderable extent.

4. When the height or diſtance exceeds the limits of the table; then, firſt, if the diſtance be given, divide it by 2, or 3, or by 4, &c. till the quotient comes within the diſtances in the table; then take out the height anſwering to the quotient, and multiplying it by the ſquare of the diviſor, that is, by 4 or by 9, or by 16, &c. and the product will be the height required. For example,—If the top of a hill can juſt be ſeen at the diſtance of 30 miles; then

30 divided by 3 gives 10, to which in the table anſwers $66\frac{1}{3}$ feet; which multiplied by the ſquare of 3 or 9, gives 597 feet for the height of the hill. But when the height is given, divide it by one of theſe ſquare numbers, viz. 4, 9, 16, 25, &c. till the quotient come within the limits of the table, then take out the diſtance anſwering the quotient, and multiply it by the ſquare root of the diviſor; that is, by 2, 3, 4, or 5, for the diſtance required. For example,—let it be required to find the diſtance the top of a mountain can be ſeen, whoſe height is 4 miles or 21120 feet; divide 21120 by 256, or the ſquare of 16, and the quotient is 83 nearly, to which in the table anſwers, by proportion, nearly $11\frac{1}{5}$ miles; then mltiplying $11\frac{1}{5}$ by 16, gives $179\frac{1}{3}$ miles for the diſtance of the mountain.

THE END.